**Theory of Error-Correcting Codes**

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**Time:** S3 / 24H CM / 2.5 ECTS  
**Location:** SUPELEC  
**Pre-requisites:** SAR-B11, SAR-C1, SAR-C21, SAR-C22  
**Grading:** Final exam (jointly with SAR-C31 and SAR-C33)  
**Homework:** Theoretical exercises and/or programming exercises

**Abstract**  
The field of channel coding started with Claude Shannon’s 1948 landmark paper. Fifty years of efforts and invention have finally produced coding schemes that closely approach Shannon’s channel capacity limit on AWGN channels, both power-limited and band-limited. Similar gains are being achieved in other important applications, such as wireless channels. This course is divided in two parts. In the first part, we remind students of the basics of the theory of linear codes for conventional memoryless ergodic channels. We then introduce more advanced notions so as to make comprehensible some of the most recent coding schemes proposed in the literature. In the second part, we expound the principles of coded modulations for the Gaussian channel and, if time permits, for Rician and Rayleigh fading channels (fully interleaved). We will conclude the course by evoking some aspects of code design for non-ergodic block-fading channels.

**Course outline**

**Lecture 1.** Fundamental limits [1, chap. 4][2, chap. 8][3]  
- Basic definitions – Classification of channels, random codes  
- Coding theorem for DMC – Upper bounds on error probability  
- Coding theorem for DMC – Lower bounds on error probability  
- Strong converse theorem for discrete channels [Wolfowitz]  
- Generalization of results to BIOS memoryless channels  
- Hard or soft decoding, information loss  
- Cut-off rate

**Lecture 2.** Codes with algebraic structures [4][5]  
- Detection and correction capabilities of block codes  
- Lower and upper bounds on code parameters – General case  
- Linear block codes – Minimum distance, duality, elementary transformations  
- Lower and upper bounds on code parameters – GV, Hamming, Plotkin, Singleton  
- Convolutional codes
- Upper and lower bounds on code parameters – Asymptotical bounds

**Lecture 3.** Graphical representations of linear codes [6]
- Bipartite graphs [Gallager, Tanner]
- Trellises – Construction and algebraic properties
- Trees

**Lecture 4.** MAP/ML decoding algorithms
- Sequence-by-sequence MAP decoding – Viterbi
- Symbol-by-symbol MAP decoding – BCJR, Hartmann
- Ordered-statistics based probabilistic decoding – Fang-Battail, Fossorier-Lin

**Lecture 5.** Performance analysis of linear codes under ML decoding [7]
- Weight distributions – Definitions, MacWilliams identity, dual codes, MDS codes
- System analysis and error processing strategies
- Upper bounding techniques – Union bound, TSB, other refined bounds
- Lower bounding techniques – De Caen’s bound, other refined bounds

**Lecture 6.** Fundamentals of turbo coding [8, chap. 4][9, chap. 10,11][10, chap. 6]
- Types of concatenations – Parallel, serial, hybrid
- Weight distributions – Parallel, serial
- Performance analysis under ML decoding
- Sub-optimum iterative decoding
- Convergence analysis using EXIT charts

**Lecture 7.** Non-binary coding for bandwidth-limited AWGN channels
- Brief introduction to lattice theory [11, chap. 1,2]
- Trellis coded modulations – Design criteria under MLD [9, chap. 3]
- Multilevel coding – Design criteria under MLD/MSD [Wachsmann]

**Lecture 8.** Space-time coding for wireless channels [12, chap. 6,7]
- Performance analysis under MLD and code design criteria
- Tarokh’s space-time trellis codes
- Bit-interleaved coded modulations

**References**